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Graphical construction for the direction of shear

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ABSTRACT

A new graphical method is developed to determine the direction of the maximum resolved shear stress on a fault plane. It differs from existing graphical methods in using the direction perpendicular to the maximum resolved shear. It is based upon the theory of vector manipulation, making the proposed method more straightforward and more graphical, and, hence, we believe more accessible.

1. Introduction

Determining the direction of the maximum resolved shear stress on a certain planar surface, such as a fault or crystallographic glide plane, plays a fundamental role in examining the onset of slip along these planes for a given stress state, and it has been vigorously addressed by many structural geologists (e.g., Johnson and Mellor, 1973; Lisle, 1989, 1998; Means, 1989; DePaor, 1990; Ragan, 1990; Fry, 1992; Fleischmann, 1992; Ritz, 1994). For the task of graphically deriving the maximum shear direction on a surface, given the principal stresses, numerous methods have been developed, according to the preferences of different authors in visual construction. Despite the fact that the shear on the surface is readily numerically determined, it is somewhat difficult to determine it directly in a graphical way. All of the existing methods were devised to look directly for the direction of the maximum resolved shear stress on the surface, each in their own way.

This short contribution is aimed at developing a new graphical method for the above task. It will be shown below that the direction of the maximum resolved shear stress can be obtained from its perpendicular direction on the surface that is graphically determined using the method proposed in this paper. The determination of the latter direction is very straightforward, using the theory of vector manipulation.

Throughout this paper, compressional stress is positive in sign, and tensional stress negative.

2. Fundamentals

Let $\boldsymbol{n}$ stand for the normal unit vector to a fault plane.

$$\boldsymbol{n} = [n_1, n_2, n_3] \tag{1}$$

where n_1, n_2 and n_3 are the Cartesian coordinates along the $\boldsymbol{X}$-axis, the $\boldsymbol{Y}$-axis and the $\boldsymbol{Z}$-axis, respectively. For the known stress ($\boldsymbol{\sigma}$), the stress vector or traction ($\boldsymbol{t}$) and the maximum resolved shear stress ($\tau_{\max}$) exerted on the plane are readily calculated according to the following expressions (e.g., Ragan, 1990; Lisle, 1998):

$$\boldsymbol{t} = \boldsymbol{n\sigma} \tag{2}$$

$$\tau_{\max} = \boldsymbol{t} - \boldsymbol{n}\left(\boldsymbol{tn}^{\mathrm{T}}\right) \tag{3}$$

where T is matrix transpose. In Eqs. (2) and (3), $\boldsymbol{t}$ is the projection of $\boldsymbol{\sigma}$ on $\boldsymbol{n}$, and the projection of $\boldsymbol{t}$ on the fault plane is $\tau_{\max}$ lying in a plane through $\boldsymbol{n}$ and $\boldsymbol{t}$. Despite the simplicity of these equations, it is rather difficult to directly determine the direction of the maximum resolved shear stress in a graphical way, for instance, by using the stereonet that is familiar to structural geologists. That is why many graphical methods for the determination (e.g., Johnson and Mellor, 1973; Lisle, 1989, 1998; Means, 1989; DePaor, 1990; Ragan, 1990; Fry, 1992; Fleischmann, 1992; Ritz, 1994) have been devised.

Let $\boldsymbol{l}$ be the unit vector on the fault plane perpendicular to the maximum resolved shear stress.

$$\boldsymbol{l} = [l_1, l_2, l_3] \tag{4}$$

where l_1, l_2 and l_3 are the Cartesian coordinates along the $\boldsymbol{X}$-axis, the $\boldsymbol{Y}$-axis and the $\boldsymbol{Z}$-axis, respectively. According to Eqs. (2) and (3),

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there is no resolved shear in the direction perpendicular to the maximum resolved shear stress. Therefore,

$$\tau_{max}\boldsymbol{l}^{T} = 0 \tag{5}$$

Inserting Eq. (3) into Eq. (5) yields

$$\boldsymbol{n}\boldsymbol{\sigma}\boldsymbol{l}^{T} = 0 \tag{6}$$

In practice, Eq. (6) is one of the most important relationships for inversion of stress from fault/slip data (e.g., Fry, 1999; Shan et al., 2003), because the fault striation is considered parallel to the direction of the maximum resolved shear stress (Carey and Brunier, 1974).

Let us define the coordinate axes such than the maximum, intermediate and minimum principal stress directions are the ***X***-axis, the ***Y***-axis and the ***Z***-axis, respectively. Eq. (6) is then simplified to,

$$n_1 l_1 \sigma_1 + n_2 l_2 \sigma_2 + n_3 l_3 \sigma_3 = 0 \tag{7}$$

where σ_1, σ_2 and σ_3 are the maximum, intermediate and minimum principal magnitudes, respectively.

By definition, unit vectors $\boldsymbol{l}$ and $\boldsymbol{n}$ are mutually perpendicular. Hence,

$$n_1 l_1 + n_2 l_2 + n_3 l_3 = 0 \tag{8}$$

Solving $n_3 l_3$ from this equation, and then inserting it into Eq. (7),

$$n_1 l_1(\sigma_1 - \sigma_3) + n_2 l_2(\sigma_2 - \sigma_3) = 0 \tag{9}$$

$$\frac{n_1}{n_2}\frac{l_1}{l_2} = -\frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} = -\phi \tag{10}$$

where ϕ is the stress ratio (Bishop, 1966). The importance of this concept lies in the fact that all stresses inverted from fault/slip data or focal mechanism data are not principal stress values but only the relative magnitudes of these. Eq. (10) is essentially similar to those equations deduced by many authors, e.g., Lisle's (1998) equation (8) and Fry's (1992) equation at step 3.

From Eq. (10), the projection of $\boldsymbol{l}$, or $[l_1,l_2]$, in the $\sigma_1\sigma_2$ plane is perpendicular to the projection of $\boldsymbol{n}$, or $[n_1,n_2]$, for $\phi = 1$, and is parallel to the intermediate principal direction when $\phi = 0$. Located in the range bounded by these two end members is the projection of $\boldsymbol{l}$ for $0 < \phi < 1$. Interestingly, this gives rise to a possible range of the maximum resolved shear stress direction on the fault plane for an unknown ϕ.

Let us rewrite Eq. (10) as

$$\frac{l_2}{l_1} = \frac{-n_1}{\phi n_2} \tag{11}$$

Therefore, simultaneous solution of Eqs. (8) and (11) can give the unit vector $\boldsymbol{l}$, from which the direction of the maximum resolved shear stress on the fault plane is readily obtained. However, instead of direct calculation, a simple graphical determination of vector $\boldsymbol{l}$ based upon these equations is described in the section below.

3. Construction

To illustrate the graphical method described in this section, the example used by Ragan (1990) will be taken (Fig. 1a). In the example, the maximum (σ_1), intermediate (σ_2) and minimum (σ_3) principal stresses have a bearing and plunge of 337°/78°, 223°/05° and 132°/11°, respectively, and the stress ratio is 0.33. For this stress, the direction of the maximum resolved shear stress on a fault plane with a dip direction of 070° and a dip angle of 60° is constructed in the following steps (Fig. 1).

Step 1 Plot on a stereogram the principal axes and the normal ($\boldsymbol{n}$) to the fault plane, and draw the great circles of the fault plane and of principal stress plane containing σ_1 and σ_2 (Fig. 1a).

Step 2 Draw a great circle through $\boldsymbol{n}$ to σ_3, and mark point $\boldsymbol{p}$ at its intersection with the $\sigma_1\sigma_2$ plane (Fig. 1b). Read along the great circle the angle (θ) between $\boldsymbol{p}$ and σ_1, 58° in this case, and mark point $\boldsymbol{q}$ perpendicular in the $\sigma_1\sigma_2$ plane to $\boldsymbol{p}$. As previously discussed, the projection of $\boldsymbol{l}$ on the principal plane is $\boldsymbol{q}$ for $\phi = 1$, and σ_2 for $\phi = 0$.

Step 3 Draw an ***X***–***Y*** graph to show points $\boldsymbol{p}$ and $\boldsymbol{q}$ projected onto the $\sigma_1\sigma_2$ plane (Fig. 1c). As in Eq. (10), the vectors of $\boldsymbol{p}$ and $\boldsymbol{q}$ might have ***X***–***Y*** coordinates $[n_1,n_2]$ and $[n_2,-n_1]$, respectively. They have non-unit length, and are mutually perpendicular. Keep in mind that we are interested in the absolute value of angles in Fig. 1c. Rescale the ***X***-coordinate of $\boldsymbol{q}$ to ϕn_2, to give point $\boldsymbol{r}$, $[\phi n_2,-n_1]$. Read the acute angle (φ) between $\boldsymbol{r}$ and σ_1, 62° in this case. It is worthwhile to note that it is not necessary to use any trigonometrical functions to solve Eq. (10) for φ, as in the preprocessing of data for some graphical methods (Fry, 1992; Lisle, 1989, 1998).

Step 4 Mark the direction of $\boldsymbol{r}$ on the stereogram at angle φ from σ_1 on the $\sigma_1\sigma_2$ plane (Fig. 1d).

Step 5 Draw a great circle through $\boldsymbol{r}$ and σ_3, and mark its intersection point $\boldsymbol{l}$ with the great circle of the fault plane. Find the direction of the maximum resolved shear stress (τ), 76°/60° in this case, that is perpendicular to $\boldsymbol{l}$ on the great circle of the fault plane (Fig. 1e).

Step 6 Identify each of the sides of the fault plane that has the end of τ within 90° of σ_1 and σ_3, and determine the shear sense by the criterion that the stress acts from the end of τ within 90° of σ_1 to the end of τ within 90° of σ_3 (Fry, 1992). The shear sense is normal for the fault in this case.

4. Discussions and conclusions

As shown above, the direction of the maximum resolved shear stress on the fault plane can be obtained from its perpendicular direction on the plane determined graphically by using the method proposed above. This method is different from all existing methods that try a variety of ways to locate the shear direction directly. It is based upon the theory of vector manipulation, and, more importantly, requires far less specialized knowledge about the stress tensor than is necessary for many existing graphical methods (e.g., Means, 1989; Fry, 1992; Lisle, 1989, 1998). Accordingly, calculation is reduced to a greater degree than any of the existing graphical methods, needing no reference to a calculator for the solution of the trigonometrical functions. All these features make the method simpler and more straightforward and even more graphical. Therefore, we believe that it would become more accessible to readers, particularly those unfamiliar with the subject.

The disadvantage of the method proposed above, if significant, is the need of a few more graphical steps. In this sense, plotting more points and great circles probably takes slightly more time and more effort in the construction.

The method developed in this paper can also be used, as an analogue, to determine the direction of shear strain. For the sake of brevity, this will not be discussed here, and interested readers are encouraged to do this by themselves.

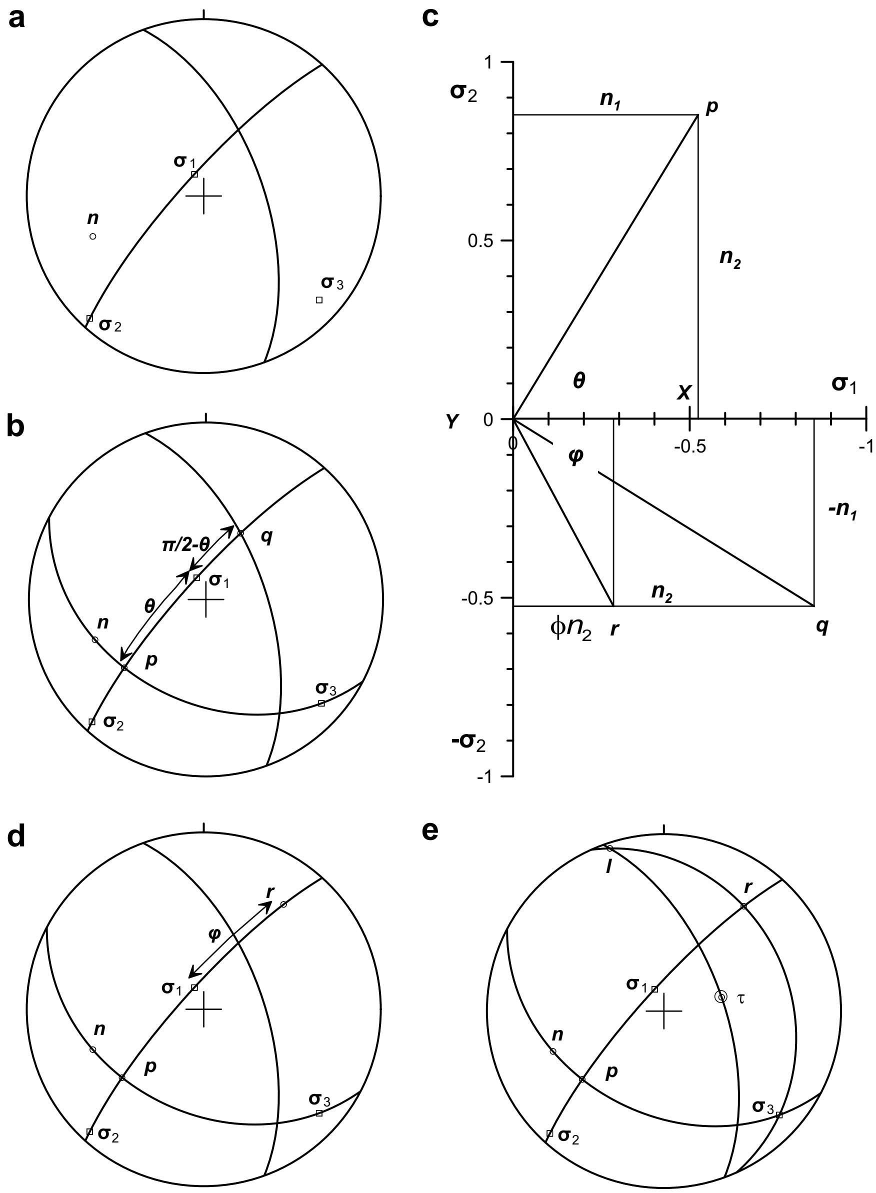

Fig. 1. Steps (a)–(e) for applying the graphical method proposed in the paper to an example taken from Ragan (1990). See the text for more definitions and more detailed explanation at each individual step. Lower hemisphere projection is used in each stereogram, at the top of which the short line is directed towards the north. In Fig. 1c, the coordinates of ***p*** and of ***q*** are scaled up to have unit length, for the sake of convenience in display. This is made justifiable by our interest only in their inclination angles with the ***X***-axis, and not the absolute values of their coordinates.

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